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Social and individual learning in a *microeconomic* framework.

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Abstract In the modeling of learning processes a distinction is generally made between individual and social learning: cognitive science studies both the functioning and the evolution of individual learning; the theory of rational social learning studies how decision makers are influenced by the choices of the others (social learning) [12]. We focus on the problems of the interactions between individual and social learning in a *microeconomic* framework. In this work agents have to choose two different modalities of investment, with different (and unknow) profitability. We show the effect of different learning procedures during the evolution and at steady state.

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1 Introduction

There are three main strands of literature that have emerged in the context of learning in economic framework: (i) adaptive procedures based on backward-looking criteria (typically imitation rules); (ii) adaptive procedures based on forward-looking criteria (for example each player keeps track of distribution of own strategies, or the values of own variables, using reinforcement learning). (iii) The third strand is the use of evolutionary procedures [10] [4]. We study the interactions between (i) and (ii) learning model.

The modeling of learning processes in the cognitive economics takes place within context of stochastic repeated game and the question of the description of the game can cause great difficulties. In this paper we use a slightly different approach. In the middle of 80's Kirman suggest the use of stochastic agents within the markets. The real take-off for the statistical mechanics model in microeconomics began in the 90' in the USA. The tools are statistical mechanics framework, mean field theory and, recently, graph theory [7] [8]. In this work we use a dynamical system approach: instead to describe with expected values the interesting quantity (average price, exchange volumes and so on), we use a deterministic system of differential equations [9] [11] [6]. The functions are the averages of how many agents are *doing something* at t time. In this approach, the interesting quantities are the behaviors. Artificial life modeling and, in general, biological inspired modeling use the behavior quantification and categorization to build computational description of their systems [1] [6] [1]. In this work, we try to translate this approach in a microeconomic context.

2 The model

In our model N agents may make a choice between two different portfolio (see figure 1). **No investment.** We define an agent that makes this choose as an agent in $Outer_{Area}$. **Investment of type A.** In this case we said that the agent is in A_{Area}. **Investment of type B.** We said that the agent is in B_{Area}). The agents modify own positions (for example from A_{Area} to $Outer_{Area}$) using two different learning procedures: **(a) individual learning**. If an agent stays in A_{Area} (or in B_{Area}) it tends to remain in the same position (if the agent knows A_{Area} tend to remain in A_{Area}, vice versa for B_{Area}; **(b) social learning**. The behavior of agents (to stay or to go) is influenced by how many agents stay in the same position at the same time.

The investment of type B is worse than the investment of type A. This information is not directly available to agents. We account that the A investment is more profitable than B investment moving ,with $\delta = Death_{Rate}$ frequency, agents from B_{Area} to $Outer_{Area}$.

[1] This work is inspired from a set of simulations of real robots. They reach different zones, the white and the black, with different *land quality*.

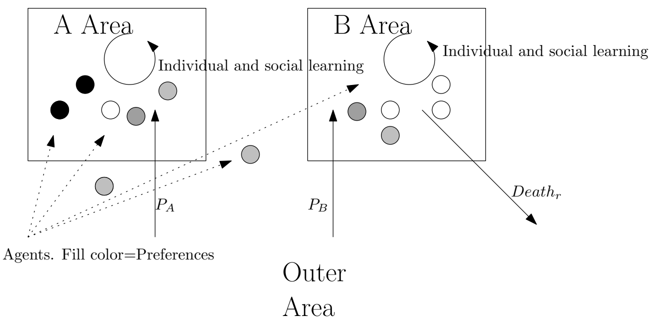

Fig. 1 The model. Agents may make a choice between three different portfolio (we said that the agents move in different areas). The color of the agents is the preference (black means A preference)

We denote by O, A and B the densities of the outer area, the A_{Area}, and the B_{Area}, respectively at time t. Those densities may be described by a gain-loss model. This technique is based on the description of the variations in terms of positive signed contribution (agents entering an area), and negative signed contribution (agents escaping an area). Indeed, at each time step an agent in the outer area may fall in the A_{Area} with probability p_A, or in the B one with probability p_B.

On the other hand, an agent in A or B_{Area} may decide whether to remain in it or exit from it, according to its acquired learning. The average of individual probabilities of remaining in A or in B areas at time t are denoted by π_A and π_B, respectively. Hence, it is straightforward to state the $1 - \pi_A$ and $1 - \pi_B$ denote the probabilities to exit from the A_{Area} and from the B one, respectively.

The parameter δ corresponds to the death rate for individuals in the black area.

We describe the dynamics of the system with an ODE system:

$$\begin{cases} \dot{O} = -(p_A + p_B)O + (1-\pi_A)A + (1-\pi_B)B + \delta B \\ \dot{A} = p_A\, O - (1-\pi_A)A \\ \dot{B} = p_B\, O - (1-\pi_B)B - \delta B \\ \dot{\pi}_A = \begin{cases} (1-\pi_A)A, & Social \\ \sigma(1-\pi_A)A\; sign(A), & Individual \end{cases} \\ \dot{\pi}_B = \begin{cases} (1-\pi_B)B, & Social \\ \sigma(1-\pi_B)B\; sign(B), & Individual \end{cases} \end{cases} \tag{1}$$

3 Results

As far as the outer area is concerned (see system (1)), we have that this density looses agents falling in a target area,

$$-p_A\, O - p_B\, O \tag{2}$$

since independent probabilities are multiplied, whereas it gains agents escaping from those areas,

$$(1-\pi_A)A+(1-\pi_B)B \tag{3}$$

and it also gains the contribution from the re-borne agents δB. With the same argument, we have that A looses $(1-\pi_A)A$ escaping agents and gains $p_A O$ from outer area (equation 2 in system (1)). In the same way, B gains $p_B O$ from outer area, but looses $(1-\pi_B)B$ escaping agents, and δB (equation 3 in system (1)). We remark here that there's no direct interaction between the two areas. The probabilities of permanence π_A and pi_B increase proportionally to the the probability of escaping that area, since it is higher when the probability of permanence is low, and vice versa. Nevertheless, those probabilities have two different factors according to the type of learning. Those factors play the role of learning rates. In the case of individual learning, as stated above, the factor σ is constant, and it is only related to the presence of agents in the area through the *sgn* function (Equations 4 and 5 in system (1)). We remark here that we assume the step function *sgn* to return 0 if the argument is zero, therefore it acts like a boolean indicator of any presence in an active area. On the other hand, in the case of social learning the factor is given by the density of the area itself, since it depends on the total number of agents in the same area.

In the figure 2, 3, 4 we show the density of area A with three different values of σ.

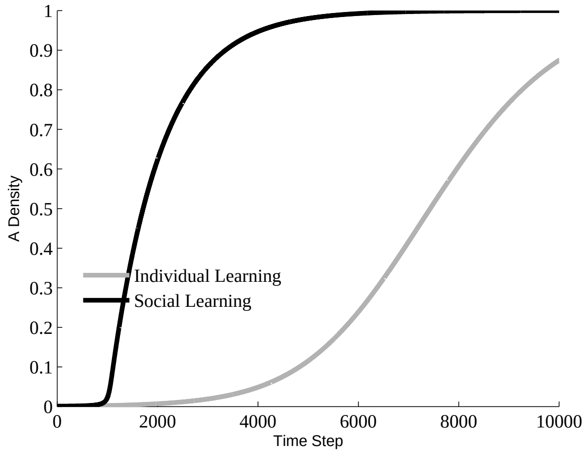

Fig. 2 Density of area A using social and individual learning with $\sigma = 0.001$

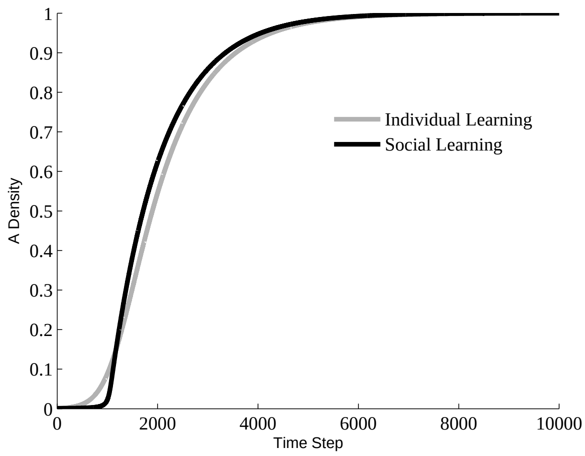

Fig. 3 Density of area A g with $\sigma = 0.005$. The individual learning curve partially overlaps the social learning curve

The ordinary differential equations stated in (1) can be used to study analytical steady states. The equations are polynomial with respect to the

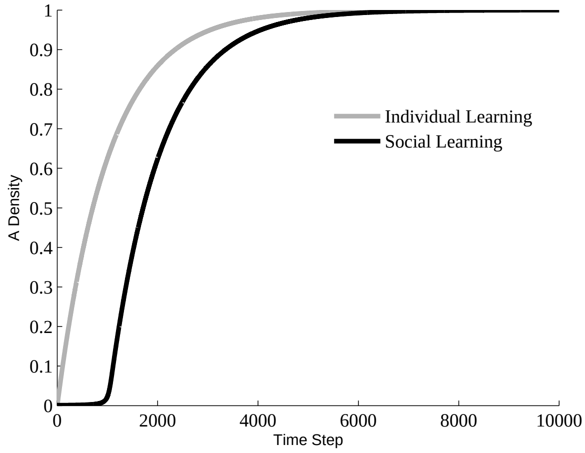

Fig. 4 Density of area A with $\sigma = 1$. The shape of the curves does not change in the range $\sigma = [0.005, \ldots, 1]$

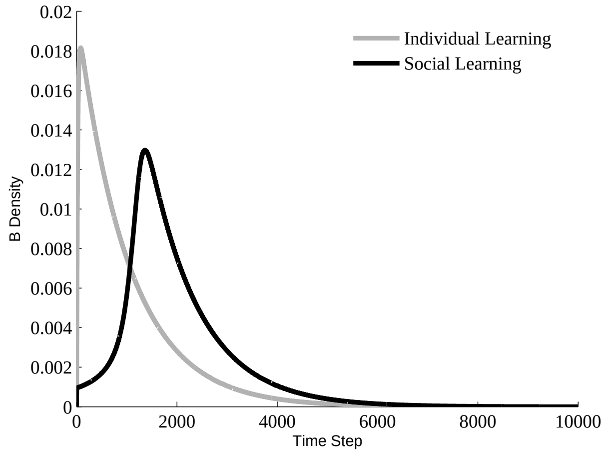

Fig. 5 Density of area B with $\sigma = 1$

unknown variables, and only two trivial states can be obtained: all the agents concentrated in one area

$$O(\infty) = B(\infty) = 0\ , A(\infty) = 1\ or\ O(\infty) = A(\infty) = 0\ ;, B(\infty) = 1$$

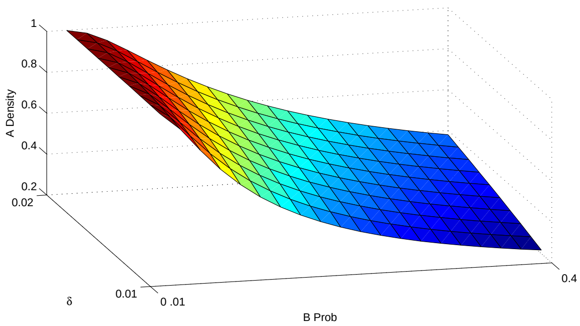

Fig. 6 Density of B area with $\delta = [0.01, \ldots, 0.02]$, $p_B = [0.01, \ldots, 0.4]$. Individual learning

Nevertheless, non-trivial states may be expected where agents are present in both the A and the B area, even for long time runs. Indeed, the results depend on the death rate σ and on the probabilities to reach the A and B areas by chance, p_A and p_B. Provided that the p_B is quite large (with respect to p_A), and δ is sufficiently small, it may happen that the density of the B area rises up to an equilibrium with respect to the A one. Some oscillations are produced by death and birth processes, hence it may happen that the two densities oscillate syncronically around an expected mean value. The corresponding behavior is an attractor-type sketch, as in the Lotka-Volterra equations for prey- predator model. These limit cycles are not included in this exposition.

In the figures 6 and 7 we show the density of area A using different value for δ and for p_B. Figure 6 shows the results using individual learning. Figure 7 shows the results using social learning.

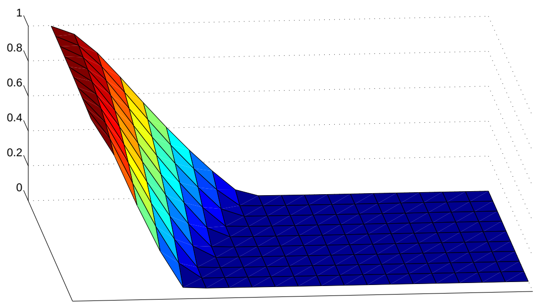

Fig. 7 Density of B area with $\delta = [0.01, \ldots, 0.02]$, $p_B = [0.01, \ldots, 0.4]$. Social learning

In figure 5 we show dynamic for density of B_{area} with $\sigma = 1$. From $t = 0$ to $t = 500$ (approximately) the figure shows that the shape of the curves are really different for individual and social learning. The figures 8 and 9 show the value of $\dot{B}$ at $t = 500$ for individual and social learning modalities, changing σ and p_B.

4 Discussion

4.1 Individual vs. social

The main results is the greater effectiveness of individual learning compared to social learning. The figure 2, 3, 4 show that, in our model, there is substantial difference between a constant rate of learning (individual) and a rate of learning that is function of densities (social). What are the implications in microeconomic modeling ? The first consideration is: financial market models (and data) give a poor confirm about this simulation results. The

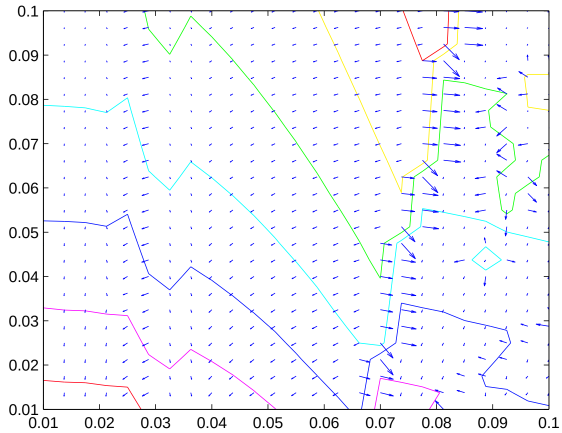

Fig. 8 $\dot{B}$ with $\delta = [0.01, \ldots, 0.1]$, $p_B = [0.01, \ldots, 0.1]$. δ is on vertical axes. Individual learning

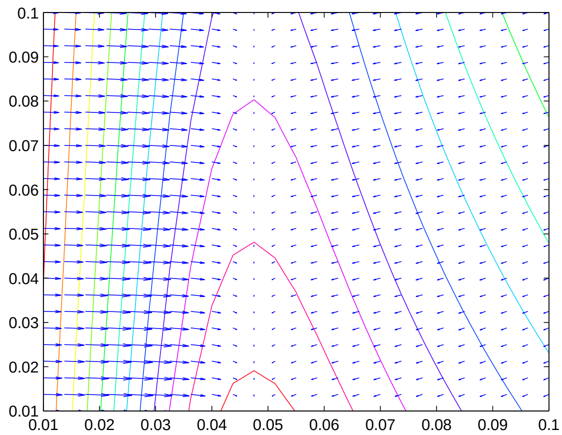

Fig. 9 $\dot{B}$ with $\delta = [0.01, \ldots, 0.1]$, $p_B = [0.01, \ldots, 0.1]$. δ is on vertical axes. Social learning

main stream of financial market models predict some kind of interaction between agents, and the statistical market data analysis lead to collective explanations about, for example, price dynamic of stocks. In our model, it seems that individual reinforcement learning lead to a final state (to choose A) faster than social learning. Besides, the individual learning reinforce the B area too: in fact each agent modifies own preferences with no information about the land quality of the area [2] [3] [5].

We guess that the explanation of this apparent paradox may be the δ dynamic. If agents with a great π_B dead without correlation with π_B, then we can imagine an indirect social effect of B area on the agents. We can explain this point with a simple biological observation. In general, *animals like what they know*: but if the thing that they like can kill them, you will observe only animals that like the *correct* thing. Animals like what they know is correct, but only alive animals can like something.

In a nutshell, the results show that the indirect social influence (by δ dynamic) provides the thrust that makes the agents move to A area.

4.2 Steady state

Can all the agents choose to avoid the *good portfolio* (A_{area})? The answer is: using the social learning, they can. In fact, with low δ and high p_B we observe $A = 0$ for a large zone of the parameters (see figure 6 and 7, in figure 7 the A density go to 0). We can see the B density dynamic in 5. We study the behavior of $\dot{B}$ when *individual learning* curve start to modify the shape (around $t = 500$). The results are showed in figure 8 and 9. They display the variation of $\dot{B}$ as a function of δ and p_B.

We notice that in social learning environment (figure 9) there is no correlation between δ values and dynamic of $\dot{B}$. The arrows are almost horizontal. The situation is different in figure 8. In this paper we avoid the description of limit cycles for the system. Anyway, from a qualitative point of view, we can said that in social learning the dynamic of $\dot{B}$ is more directed than in individual learning. This fact could (partially) explains the results in figure 7.

4.3 Developments

Actually, an open problem in microeconomic model is the complete explanation for cluster volatility of the financial markets. Empirical data show that the volatility of a stock value (i.e. his variance) is not constant during the time, but is clustered in blocks. There are many explanations (and predictive models) for this phenomena; they are based on statistical interpretations of interactions between agents during the evolution of stock price.

With this paper we suggest to use a deterministic (vs. statistical) methods to model the interactions. This work is an initial step in that direction.

Now, the system (1) contain constant rate for moving probabilities. You can imagine to substitute p_A and p_B with periodic functions of time. We are completing a general study about limit cycles for low δ with periodic p_A and p_B. If some results would show the presence of quasi-periodic behavior, it leads to interesting consideration about the dynamic of interacting agents.

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